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Structurally dynamic spin market networks

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The agent-based model of stock price dynamics on a directed evolving complex network is suggested and studied by direct simulation. The stationary regime is maintained as a result of the balance between the extremal dynamics, adaptivity of strategic variables and reconnection rules. The inherent structure of node agent "brain" is modeled by a recursive neural network with local and global inputs and feedback connections. For specific parametric combination the complex network displays small-world phenomenon combined with scale-free behavior. The identification of a local leader (network hub, agent whose strategies are frequently adapted by its neighbors) is carried out by repeated random walk process through network. The simulations show empirically relevant dynamics of price returns and volatility clustering. The additional emerging aspects of stylized market statistics are Zipfian distributions of fitness.

Keywords: econophysics, complex networks, agent-based model

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1. Introduction

The design of the decentralized systems of autonomous interacting agents with abilities to automatically devise societies so as to form modules that accommodate their behavior via social and economic norms in emergent ways is highly challenging. Agents in such groups plan and act in a way to increase their own utility.

As a general theoretical framework for starting of the statistical consideration of interacting agent entities we take into account the Ising model. Being made simply by binary spin variables, the model is able to reproduce different complex phenomena in different areas of science like biology [1], sociology [2] economy [3,4,5] or informatics [6].

Looked at from the perspective of the economics this example has a great importance because it demonstrates that a basic interaction between the spins (agents)

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can bring non-trivial collective phenomena. The parallels between fluctuations in the economic and magnetic systems afford an application of spin models to the market statistics [7,8,9]. The attempts [10,11,12,13,14] examine a context with the *minority game* theory [16]. Basic terms borrowed from the magnetic systems have been built in to the spin agent concept: exchange interaction between spins and interaction with random field. The approaches based on the minority game assume competition of the short-range ferromagnetic and global magnetization terms that are crucial for modeling of bubbles and crashes. The justification for our present formulation comes from [18]. It assumes that each among $i = 1, 2, \ldots, L$ interacting traders (agents) owns her/his regular lattice site position i and a corresponding spin variable $S^{(t)}(i) \in \{-1, 1\}$, where the upper index (t) labels the market time. Each agent, has an attitude to place buy order $S^{(t)}(i) = 1$ or to place sell order described by $S^{(t)}(i) = -1$. The variable $S^{(t)}(i)$ is updated by an asynchronous stochastic heat-bath dynamics expressed in terms of the *local effective field*.

The collective effect of spins is characterized by the instant magnetization

$$m^{(t)} = \frac{1}{L} \sum_{i=1}^{L} S^{(t)}(i) \tag{1}$$

that is interpreted as measure price imbalance. Therefore the logarithmic price return can be written as [17]

$$\ln \left[p^{(t+1)} / p^{(t)} \right] = m^{(t)} / \lambda \,, \tag{2}$$

where λ is the liquidity constant. The predominance of buy orders manifests itself throughout $m^{(t)} > 0$. In that case the instant price $p^{(t+1)}$ exceeds $p^{(t)}$. In an analogous manner $m^{(t)} < 0$ describes the fall of the stock price.

In the spin models the non-Gaussian contribution to distribution of magnetization is formed due to spin-spin interactions. The realistic models of social/economic networks can serve as topological substrate for spreading of the interaction effects. Our recent approach to spin market models [19] has been formulated for network geometry. The attempt to postulate dynamics of network that is coupled to the spin degrees of freedom is analogous to *structurally dynamic cellular automata* [20] in which the conventional cellular-automata rules are generalized to formalism where geometry and matter are dynamically coupled.

Since our former attempt did not give satisfactory topological results, one of the aims of our present work is to propose richer and more reliable formulation.

The main interests of our present research is focused on:

(1) An alternative protocol of the network reconnection assuming the network segmented by local "authorities"(local leaders - hubs).
(2) The suggestion of rules that support formation of the "small world" and "scale-free" paradigms for social networks.
(3) The including of the small-scale intra-agent cognitive-like sensorial structures sustained by interactions within the complex network of agents.

The plan of the paper is as it follows. In the next section 2 we discuss the basic network properties. The overall dynamics are given in section 3. A more detailed definition of the model items appears in subsections 3.1-3.5. In section 4 we present statistical characteristics extracted from our simulation.

2. The network topology

Let's suppose that the market structure is determined by the underlying complex network defined by dynamical rules for active links between agents. Consider the directed network (graph) of labeled nodes $\Gamma = \{1, 2, \ldots, L\}$, where node $i \in \Gamma$ attaches via $N_{\rm out}$ directed links to its neighbors $X_n(i) \in \Gamma$, $n \in I_{\rm out} \equiv \{1, 2, \ldots, N_{\rm out}\}$, i.e. the graph is $N_{\rm out}$-regular. Two outgoing links $X_1(i) = 1 + i \bmod L$, $X_2(i) = 1 + (L + i - 2) \bmod L$ of node $i \in \Gamma$ create the bidirectional cycle *static subgraph* (L-gon). The reconnection rules are applied exceptionally to the links $X_n^{(t)}(i)$, $n \in \overline{I}_{\rm out} = \{n_1; 3 \leq n_1 \leq N_{\rm out};\, n_1 \in I_{\rm out}\}$. The using of static module ($\{1, 2\} \subset I_{\rm out}$) guarantees the preservation of network connectedness at any stage t.

3. The formalism of co-evolutionary dynamics

Formally, the stochastic co-evolutionary dynamics of agents can be described by the recursive formula

$$\overline{\Pi}^{(t+1)} = \hat{\mathbf{U}}(\,\overline{\Pi}^{(t)}) \tag{3}$$

including the composed configuration

$$\overline{\Pi}^{(t)} \equiv \left\{ \Pi^{(t)}(1), \Pi^{(t)}(2), \ldots, \Pi^{(t)}(L) \right\} \tag{4}$$

that consists of single-agent particulars

$$\Pi^{(t)}(i) \equiv \begin{cases} \text{intranet intra-agent spins} & \Pi^{(t)}_{\rm ss}(i) \equiv \{s^{(t)}(i,k)\}_{k\in\Gamma_{\rm intr}} \\ \text{strategic variables} & \Pi^{(t)}_{\rm J}(i) = \{J^{(t)}_{\rm intr}(i,k,q)\}, \\ & \text{where } k, q \in \Gamma_{\rm intr} \\ \text{network links} & \Pi^{(t)}_{\rm X}(i) \equiv \{X^{(t)}_n(i)\}\,,\ n \in I_{\rm out}\,. \end{cases} \tag{5}$$

The intranet of agent i includes fully connected recurrent network consisting of nodes $\Gamma_{\rm intr} \equiv \{1, 2, \ldots, N_{\rm intr}\}$ nodes [for more details see modification Eq.(11)]. The nonlinear operator $\widehat{\mathbf{U}}$ entails the overall effect of the following single-agent operators

$$\begin{array}{lll} \text{local field} & \widehat{U}_{\rm ss}(i) & \text{acting on } \Pi^{(t)}_{\rm ss}(i) \\ & \widehat{U}_{\rm Ad}(i_{\rm a}) & \text{acting on } \Pi^{(t)}_{\rm J}(i_{\rm a}) \\ \text{reconnection} & \widehat{U}_{\rm Re}(i_{\rm r}) & \text{acting on } \Pi^{(t)}_{\rm X}(i_{\rm r}) \\ \text{extremal dynamics} & \widehat{U}_{\rm Ex}(i_{\rm minF}) & \text{acting on } \Pi^{(t)}_{\rm ss}(i_{\rm minF})\,, \Pi^{(t)}_{\rm J}(i_{\rm minF}) \end{array} \tag{6}$$

where $i_{\rm minF}$ is defined by minimum of fitness [see Eq.(13) in further]

$$F(i_{\rm minF}) = \min_{j\in\Gamma} F(j)\,. \tag{7}$$

The pseudo-code corresponding to dynamics Eq.(3) is described in the next five steps:

- **loop I** over the Monte Carlo steps per network node
  loop II over the network of agents
  1. pick agent i randomly
  2. perform the field and spin update $\widehat{U}_{\rm ss}(i)$ [see Eqs.(8) and (9) of subsection 3.1].
  3. pick two agents randomly: $i_{\rm a}$ for the adaptivity rule and $i_{\rm r}$ for the reconnection rule.
  4. apply $\widehat{U}_{\rm Ad}(i_{\rm a})$ with probability $P_{\rm Ad}$ (see subsection 3.3)
  5. apply $\widehat{U}_{\rm Re}(i_{\rm r})$ with probability $P_{\rm Re}$ (see subsection 3.4)

  end of **loop II**
  apply the extremal dynamics $\widehat{U}_{\rm Ex}(i_{\rm minF})$ to agent $i_{\rm minF}$ having the smallest fitness within the entire system.
  end of **loop I**

After it the following steps are carried out:

- (i) store instant m calculated according Eq.(1),
- (ii) calculate the *price return* Eq.(2) and volatility as $|m^{(t)}|$
- (iii) update *fitness* $F(i)$ for all agents according Eq.(13) (see subsection 3.2) .

3.1. $\widehat{U}_{\rm ss}$: *decision making via local field, interface between extranet and intranet*

Through links $X_n(i)$ agent i gains the game-relevant information about the external nearest world. For directed network topology we define the integral spin value

$$S_{\rm nn}(i) \leftarrow \frac{1}{N_{\rm out}} \sum_{n\in I_{\rm out}} S(X_n(i))\,. \tag{8}$$

The intra-agent idiosyncratic structure is modeled in a abstract way that resembles the modular organization of the routing control unit [6]. The present variant of the model goes essentially beyond the elementary single-spin intra-agent description. We have looked in this direction by considering trader's states encoded by Ising spins (neurons) coupled by the weighted links of the *fully interconnected intranet*. We suppose the agent's i architecture consisting of $N_{\rm intr}$ spins $\{s(i,k)\}_{k\in\Gamma_{\rm intr}}$, $s(i,k) \in \{-1,1\}$. In further, we distinguish between the inner $s(\cdot,\cdot)$ and extranet

$S(\cdot)$ symbols. The effective local field $h_{\rm loc}(i,k)$ is suggested in the form

$$h_{\rm loc}(i,k) \leftarrow h_{\rm stoch}(i,k) + \frac{1}{N_{\rm intr}-1} \sum_{\substack{q \neq k \\ q,\, k \,\in\, \Gamma_{\rm intr}}}^{N_{\rm intr}} J_{\rm intr}(i,k,q) s(i,q)\,, \tag{9}$$

where $h_{\rm stoch}(i,k)$ is the Gaussian stochastic variable (i.e. the source of local mutations) and $J_{\rm intr}(i,k,q)$ is the $N_{\rm intr} \times N_{\rm intr}$ system of the intra-agent pair couplings (weights); the term $1/(N_{\rm intr}-1)$ is included for normalization reasons. Three inputs of intranet are supposed:

$$s(i,3) \leftarrow m\,, \qquad s(i,4) \leftarrow S(i)\,, \qquad s(i,5) \leftarrow S_{\rm nn}(i)\,. \tag{10}$$

Two of the spins $s(i,k)$, $k=3,5$ are linked to outer complex network represented by m and $S_{\rm nn}(i)$. The feedback to $s(i,4)$ is related to self-control. The state of kth unit of agent i is recalculated according

$$s(i,k) \leftarrow {\rm sign}\left(h_{\rm loc}(i,k)\right)\,, \qquad \text{when} \qquad k \in \overline{\Gamma}_{\rm intr} = \{1,2\} \cup \{6,7,\ldots,N_{\rm intr}\}\,. \tag{11}$$

The remark here is that recursive spin update of intranet is performed asynchronously. Two-neuron output of intranet has been considered: $k \in \{1,2\}$, [see Eq.(9)]. The agent's sell buy order

$$S(i) \leftarrow \frac{1}{2}\left[s(i,1) + s(i,2)\right] \tag{12}$$

can be identified by other agents. The factor $1/2$ normalizes the state space $\{-1,0,1\}$ of $S(i)$. The $S=0$ state is interpreted as a *passive* [14]. The impact of local $S_{\rm nn}(i)$ and global m in Eq.(10) may be interpreted as Keynes' beauty contest [15] according which the stock market notably reflects the mass psychology.

The agent's decision sell or buy is given by the procedure:

procedure $\hat{U}_{\rm ss}(i)$

loop III over $N_{\rm intr}$ repetitions

1. pick $i \in \Gamma$ randomly;
2. calculate $S_{\rm nn}(i)$ according Eq.(8);
3. make settings Eq.(10);
4. choose randomly intranet spin $k \in \overline{\Gamma}_{\rm intr}$;
5. calculate the local field at k according Eq.(11);
   the stochastic field $h_{\rm stoch}(i,k)$ is calculated for each node separately;
6. update $S(i)$ according Eq.(12);

end of **loop III**

The **output** of the procedure $\hat{U}_{\rm ss}$ is $S(i)$ variable which can be identified by other linked agents [see Eq.(8)].

3.2. *Fitness $F(i)$ concept*

Biological species interact with each other in ways which either increase or decrease their fitness. The concept is generalizable to survival of strategies undergoing selection of the economic agents. The *extremal dynamics* [see Eq.(7)] governing co-evolution is based on the knowledge of the local fitness $F(i)$ that expresses an ability/inability to survive in the competitive environment. The selection and survival according fitness has been used for spin market models [10]. The fitness-dependent link formation [22,24,25] is conceptually close to our present formulation. In our model, the local fitness is defined as the integral over the history of agent's gains and losses calculated from market situation and sell buy orders

$$F^{(t+1)}(i) = F^{(t)}(i) + S^{(t)}(i)\left[-c_0 m^{(t)} + c_{\mathrm{rand}}\,\mathrm{N}^{(t)}(0,1)\right] . \tag{13}$$

The formulation is based on the combination of *minority game* profit and external influences. The relationship of individual spin S and majority is quantified by the $-c_0 S^{(t)}(i) m^{(t)}$ term with $c_0 > 0$. The impact of exogenous news [26] is included via random Gaussian term (white noise) $c_{\mathrm{rand}} N^{(t)}(0,1)$ that is common for all agents.

3.3. *Adaptivity procedure $\widehat{U}_{\mathbf{Ad}}(i_{\mathrm{a}})$, herding*

Many mathematical models in the social sciences assume that humans can be described as "rational" entities. Nevertheless, the most people are only partly rational and in fact are emotional/irrational in their decisions. This property is incorporated into the concepts of *bounded rationality* [27] and *herding* of agents. As a concrete example can serve the followers with their believe that imitation of given strategy owned by the social neighbors (selected with care according to their fitness) would bring a future benefit to them. The assumes the transfer of information in format Π_{J} along the edge of extranet. The adaption of i_{a} starts with the random pick of $n_{\mathrm{a}} \in I_{\mathrm{out}}$ that checks a *prototype node* $i_{\mathrm{prot}} = X_{n_{\mathrm{a}}}(i_{\mathrm{a}})$ among the nearest neighbors. It should be stressed that links are placed by requesting fitness preferences, therefore, the adaptation is random only in the sense that it is not directed to specific n_{a}.

The adaption of $\Pi_{\mathrm{J}}(i_{\mathrm{a}})$ to $\Pi_{\mathrm{J}}(i_{\mathrm{prot}})$ is described by updates

$$J_{\mathrm{intr}}(i_{\mathrm{a}},k,q) \leftarrow J_{\mathrm{intr}}(i_{\mathrm{a}},k,q)(1-\eta) + \eta J_{\mathrm{intr}}(i_{\mathrm{prot}},k,q)\,, \tag{14}$$

where plasticity parameter $\eta \in (0,1)$ expresses how quickly the follower i_{a} learns a strategy. The repeated application of $\widehat{U}_{\mathrm{Ad}}(i_{\mathrm{a}})$ yields entropy consumption and strategic uniformization of market that works against distinctive thinking.

3.4. *Reconnection rules $\widehat{U}_{\mathbf{Re}}(i_{\mathrm{r}})$, network dynamics*

Recently, the interest in complex networks has been extended to the seeking for the local rules governing the build-up of social and technological networks. Several

principles have been exploited for this purpose. As an example we mention the network that shows marks of age [28] or inter-agent communication across the net [29,30]. The core of these methodologies rely on the particular mechanism of *preferential attachment* suggested by Barabási and Albert [31]. A principal distinction from mentioned work is that our present study directs attention on stationarity conditions and constant vertex number.

Let us turn attention to our former proposal of centralized *single leader model* [19]. Intuitively, it is implausible that $L-1$ followers can continuously identify the leader and its strategy can be adapted by them in the limit of very large L. The reason for the revision of single leader picture is that the larger the market is, a more demanding and time consuming is the technical analysis of the follower that claims to localize socially attainable local leader. Accordingly, we are interested in a more general multi-leader stationary society influenced by a varying group of irregularly distributed *local leaders*.

This paper aims to suggest indirect method of generation of *segmented market* which includes many competing local leaders. The network is reconstructed indirectly via the knowledge extracted from the *random walk process on the net* [32,33]. The walk is carried out by the "assistant/informant" agent several times emitted from the same source node $i_{\rm r}$. The informant behaves like the Web surfer reading pages, jumping from one to another by clicking randomly on web links. To be more precise, we define the notion of *repeated random walk* as a set of linked nodes $\mathbf{RRW}(i_{\rm r}, N_{\rm rep}, N_{\rm depth}) \subset \Gamma$. The set is obtained by performing $N_{\rm path}$ steps that are $N_{\rm rep}$ times repeated from the origin $i_{\rm r}$ occupied by the agent seeking for a nearby local leader. The fitness ranked by walker - informant allows to propose preferential attachments that are expected to yield formation of *segmented market* with impediments to the free flow of information. The procedure of *reconnection* to a local leader denoted as $\widehat{U}_{\rm Re}(i_{\rm r})$ is build upon the sub-procedure of edge pruning $\widehat{U}_{{\rm Re}_{\rm w}}(i_{\rm r}, i_{\rm B})$ that calls the *best connection proposal* $\widehat{U}_{{\rm Re}_{\rm B}}(i_{\rm r})$. The sub-procedures are specified in bellow:

procedure $\widehat{U}_{{\rm Re}_{\rm B}}(i_{\rm r}) = i_{\rm B}$

(**1**). **loop IV** over $N_{\rm rep}$ repetitions
that start from the initial condition $i_1 \equiv i_{\rm r}$
loop V the execution of $N_{\rm depth}$ iterations

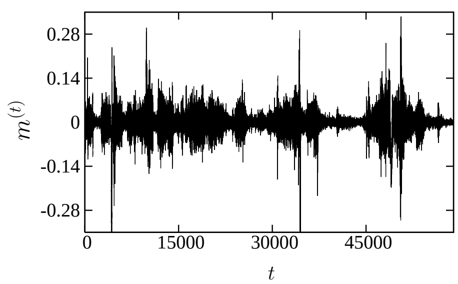
$$i_{l+1} = X_{n_l}(i_l)\,, \qquad l = 1, \ldots, N_{\rm depth}\ , \tag{15}$$

for random choices links $n_l \in I_{\rm out}$
end of **loop V**;
end of **loop IV**;

(**2**). the *comparison* of $N_{\rm depth} \times N_{\rm rep}$ values of $F(i_l)$

(**3**). and *localization* of the agent $i_{\rm B}$ according

$$F(i_{\rm B}) = \max_{i_l \in \mathbf{RRW}} F(i_l) \tag{16}$$



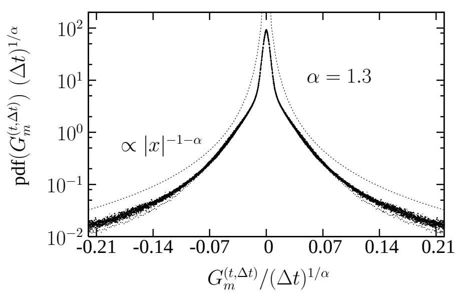


Fig. 1. (a) The time evolution of the log-price returns defined as magnetization $m^{(t)}$. Part (b) shows the scaling properties of the pdf of cumulated returns $G_m^{(t,\Delta t)} = m^{(t+1)} + m^{(t+2)} + \ldots + m^{(t+\Delta t)}$. Parameter $\alpha = 1.3$ determined at first from the fit well coincides with the scaling collapse of eight distributions obtained for trading time separations $\Delta t = 1, 2, \ldots, 8.$).

via the set $\mathbf{RRW}(i_{\mathrm{r}}, N_{\mathrm{rep}}; N_{\mathrm{depth}})$ bounded by the radius N_{depth}
formed by nodes $i_1, i_2, \ldots, i_{N_{\mathrm{depth}}}$ visited according
loops **IV**, **V**.

The **output** of $\widehat{U}_{\mathrm{Re_B}}(i_{\mathrm{r}})$ is agent $i_{\mathrm{B}} \in \Gamma$ that is the *candidate* for future connection from node i_{r}.

The pruning is performed according:
procedure $\widehat{U}_{\mathrm{Re_w}}(i_{\mathrm{r}}, i_{\mathrm{B}}) = \widehat{U}_{\mathrm{Re_w}}(i_{\mathrm{r}}, \widehat{U}_{\mathrm{Re_B}}(i_{\mathrm{r}})) = n_{\mathrm{w}}$

loop VI over the N_{out} nearest neighbors. Determine the "weakest" (*worst*) connection n_{w} with the smallest fitness

$$F(X_{n_{\mathrm{w}}}(i_{\mathrm{r}})) = \min_{n \in \overline{I}_{\mathrm{out}}} F(X_n(i_{\mathrm{r}})) \tag{17}$$

within the actual existing connections

end of **loop VI**.

The **output** of $\widehat{U}_{\mathrm{Re_B}}(i_{\mathrm{r}})$ is the index $n_{\mathrm{w}} \in \overline{I}_{\mathrm{out}}$.

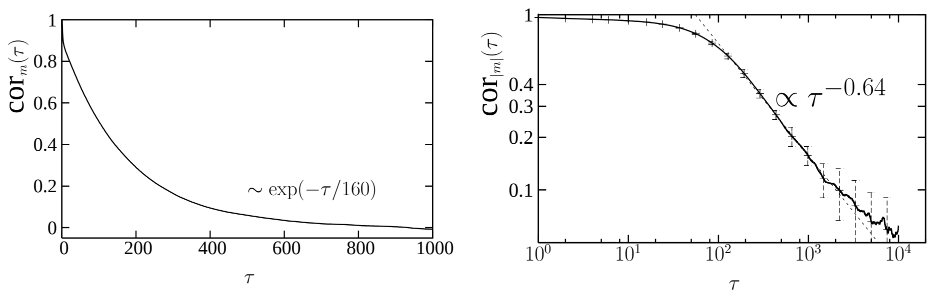
Finally, if $F(X_{n_{\mathrm{w}}}(i_{\mathrm{r}})) < F(i_{\mathrm{B}})$ the **output** of $\widehat{U}_{\mathrm{Re}}(i_{\mathrm{r}})$ is the update

$$X_{n_{\mathrm{w}}}(i_{\mathrm{r}}) \leftarrow i_{\mathrm{B}} \tag{18}$$

conditioned by the requirement that no multiple connections between i_{r} and i_{B} are established. It also forbids self-connection loops $(X_n(i_{\mathrm{r}}) = i_{\mathrm{r}})$.

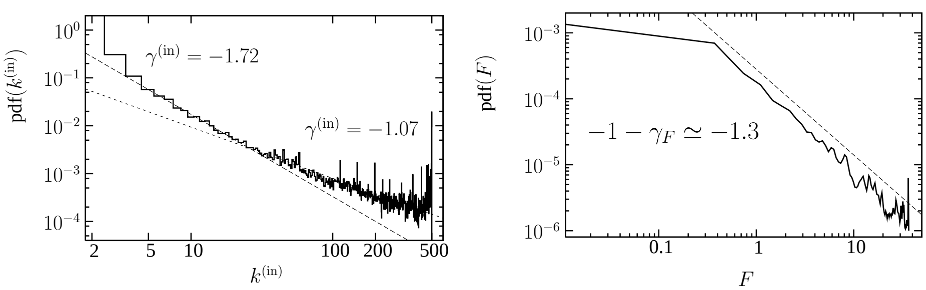
3.5. *Extremal dynamics* $\widehat{U}_{\mathbf{Ex}}(i_{\mathbf{minF}})$*, entry and exit of market*

The extremal dynamics [34,35,36] when interpreted as a bankruptcy of firm or exit of strategy from market could be considered to be the principal mechanism of the economic *co-evolution*. The idea of Bak-Sneppen model can be easily converted to



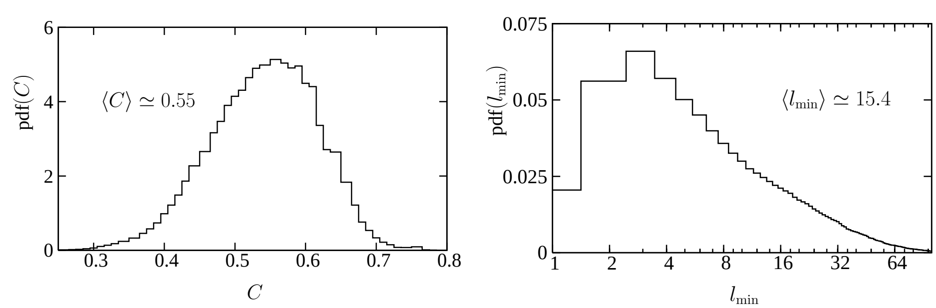
(a) (b)

Fig. 2. (a) The autocorrelation function of the logarithmic price return $m^{(t)}$. (b) The indication of the power-law long-time regime of the autocorrelation function of log-price volatility $|m^{(t)}|$.



(a) (b)

Fig. 3. (a) The power-law pdf($k^{(in)}$) with local effective exponents $\gamma^{(\mathrm{in})} = -1.07$ (when $k^{(\mathrm{in})} \geq 20$), and $\gamma^{(\mathrm{in})} = -1.07$ [for $k^{(\mathrm{in})} \in (5, 20)$]. (b) The Zipf's $\gamma_F = 0.3$ law of fitness.



(a) (b)

Fig. 4. (a) The pdf of clustering coefficients. In part (b) the pdf of the minimum path ways between randomly selected pair of nodes is plotted.

strategic variables. In that case the least fit strategy developed by i_{minF} is replaced by a random candidate from certain limited strategic space. No reconnections are assumed within the procedure $\widehat{U}_{\mathrm{Ex}}$ itself. This simplification is justified by the as-

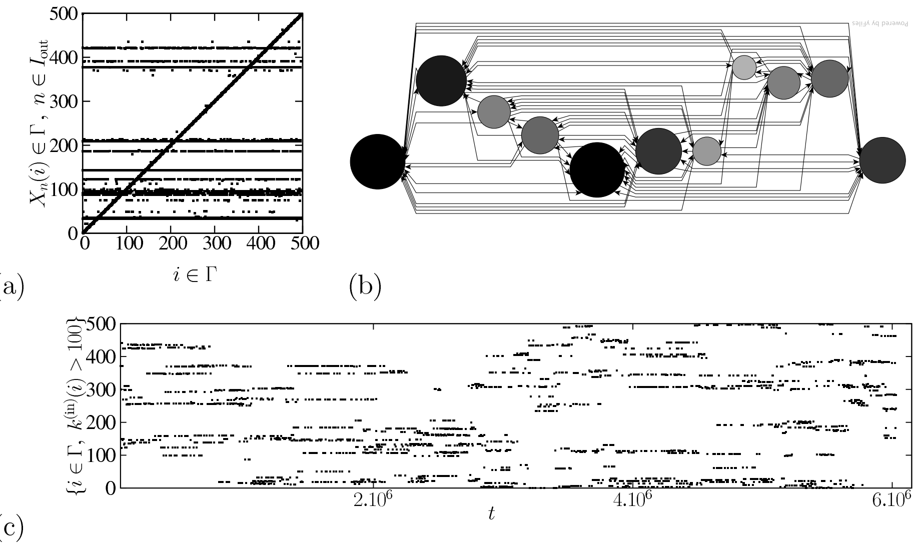


Fig. 5. (a) The snapshot of adjacency $L \times L$ matrix (the point denotes the connection). The cyclic L-gon maps onto the matrix diagonal. (b) The subgraph $\{i \in \Gamma, k^{(\rm in))} > 50\}$ (the larger circle belongs to larger $k^{(\rm in)}$. (c) The epochs of topology monitored by means of selection of highly preferred $\{i, i \in \Gamma,\, k^{(in)}(i) > 100\} \subset \Gamma$. The members of such group may be vaguely identified as local leaders.

sumption that low fitness agents are rarely attached [i.e. $k^{(\rm in)}(i_{\rm minF})$ is relatively small]. This reluctance against attachment follows from the preferences incorporated into $\widehat{U}_{\rm Re}(i_{\rm r})$. The extremal event in the present formulation means that instant value of the strategic variable is immediately replaced by Gaussian distributed random number $\mathrm{N}(0, \sigma_{\{\ldots\}})$ of dispersion $\sigma_{\{\ldots\}}$. The new strategy enters thought

$$\begin{aligned} J_{\rm intr}(i_{\rm minF}, k, q) &\leftarrow \mathrm{N}(0, \sigma_{J_{\rm intr}})\,, \\ h_{\rm stoch}(i_{\rm minF}, k, q) &\leftarrow \mathrm{N}(0, \sigma_{h_{\rm stoch}})\,, \\ F(i_{\rm minF}) &\leftarrow \mathrm{N}(0, \sigma_{\rm F})\,. \end{aligned} \tag{19}$$

These updates necessitate the fixing of three independent dispersion parameters $\sigma_{J_{\rm intr}}$, $\sigma_{\rm F}$, $\sigma_{\rm h_{stoch}}$.

4. Simulation results

4.1. *Selection of parameters*

The statistics as presented in the next subsection is determined by the choice of 13 free parameters. Their adjustment is a nontrivial task as whenever in the modeling of agency. To attain at least qualitative agreement with current statistical concepts in finance [21], the values (see Table 1) have been suggested by the optimization in the parametric space. A particular requirement is to keep the spin dynamics

Table 1. List of 13 numerical parameters for which the statistics is presented.

number of	extranet nodes	$L = 500$
	node outputs	$N_{\text{out}} = 10$
	random search steps	$N_{\text{depth}} = 6$
	repeated net searches	$N_{\text{rep}} = 6$
	intranet nodes	$N_{\text{intr}} = 8$
dispersion of	J_{intr}	$\sigma_{J_{\text{intr}}} = 1$
	h_{stoch}	$\sigma_{h_{\text{stoch}}} = 0.001$
	F	$\sigma_{\text{F}} = 0.1$
probability of	reconnection	$P_{\text{Re}} = 0.01$
	adaptive move	$P_{\text{Ad}} = 0.2$
adaptivity	parameter	$\eta = 0.1$
fitness	minority game	$c_0 = 1$
	news	$c_{\text{ran}} = 0$ except Tab.2

Table 2. The study of unexpected information about economic performance or political situations incorporated into fitness. The comparison of averages corresponding to two combinations of parameters c_0, c_{rand} of local fitness Eq.(13). The measure $P_{m,4}$ denotes the probability that $m^{(t)}$ does not alter sign during the four subsequent steps $m^{(t)}$, $m^{(t+1)}$, $m^{(t+2)}$, $m^{(t+3)}$ (four times bearish or four times bullish stock). The table shows that risk aversion accompanied by the probable passive states with reduced $\langle|m^{(t)}|\rangle$ can issue from the high exogenous influence. The calculation of $P_{m,4}$ indicates that news can break bearish (bullish) sequences. The population of local leaders quantified by exceptionally connected nodes characterized by probability $P_{k>100} \equiv \sum_{k^{(\text{in})}>100} \text{pdf}(k^{(\text{in})})$.

c_0	c_{rand}	$\langle\vert m^{(t)}\vert\rangle$	$P_{m,4}$	$P_{k>100}$	$\langle C\rangle$	$\langle l_{\min}\rangle$
1	0.00	0.0138	0.226	0.02	0.55	15.2
1	0.05	0.0082	0.165	0.02	0.56	18.4
1	0.10	0.0029	0.091	0.02	0.57	15.2

much faster than . An additional constraint aims at attaining vicinity of a *critical regime*, where the power-law distributions are obeyed [22] that are much studied topics inside Econophysics. Various mechanisms of the generation of power-laws have been recently summarized in review article [23].

4.2. *Distributions and averages*

Starting from random initial condition, we let the system evolve at least 5.10^4 Monte Carlo steps per node. The data for calculation of averages of interest have been collected from the next $\sim 10^7$ Monte Carlo steps. Their validity is verified for several independent trials. The comparison of selected averages corresponding to different fitness is given by Tab.2. The table indicates that external news yield more careful behavior of agents and preference of passive states $S = 0$ that anomalously sharpen the central part of probability density function (pdf) of m denoted as pdf(m). For

given example, the impact of $c_{\rm ran}$ on the topology is marginal.

In further we concentrate on the case $c_{\rm rand} = 0$. The statistical treatment (see Fig.1) leads to the fat tailed pdf(m) that have been fitted in particular by the power law asymptotics $|m|^{-1-\alpha}$ of the realistic effective exponent $\alpha \simeq 1.3$ [21]. The dynamical features of the price dynamics are highlighted by the autocorrelation functions in Fig.2. The clustering of the volatility $|m^{(t)}|$ of the log-price returns is observed. In that case the power-law indicates the occurrence of the long time memory. Despite of the fact that exponent -0.64 does not strictly reproduce empirical findings -0.34 [37] and -0.2 [38], the recovery of log-slope from $(-1, 0)$ range is quite encouraging for the perspectives of the model. The situation is even more interesting since the power-law pdf(F) [see Fig.3(b)] histogramed for $F > 0$ can be interpreted as Zipf's $\gamma_{\rm F} = 0.3$ law for survival abilities of strategies.

Let us turn attention to the issues of network statistics depicted in Figs.3(a),4 and 5. The *node degree* $k^{(\rm in)}(j) = \sum_{i\in\Gamma} \sum_{n\in I_{\rm out}} \delta_{j,X_n(i)}$ accounts for incoming links of node j. The stationary regime generates sequence of networks with broad-scale pdf of node degrees. By fitting of pdf($k^{(\rm in)}$) we have identified two partial effective exponents $\gamma^{\rm in} \simeq -1.07$ [for $k^{(\rm in)} \in (5, 20)$], and -1.72 (as $k^{(\rm in)} \leq 20$) from the law pdf($k^{\rm in}$) $\sim [k^{(\rm in)}]^{\gamma^{\rm in}}$. Several real exponents are provided here for illustrative purposes. The value 1.81 [40] corresponds to the collection of e-mail addresses. The exponent -1.2 belongs to the coauthorship network [41].

In further the organization of the network is done using *clustering coefficient* $C(i)$ [42] that is a local measure of interrelatedness of triplets or *social transitivity* [43]. For directed network the tendency can be measured by $C(i) = e(i)/(N_{\rm out}(N_{\rm out}-1))$, where $e(i) \equiv \sum_{n_1,n_2,n_3=1}^{N_{\rm out}\times N_{\rm out}\times N_{\rm out}} \delta_{X_{n_1}(X_{n_2}(i)),X_{n_3}(i)}$ stands for the number of the links between neighbors $X_{n_1}(X_{n_2}(i))$ and $X_{n_3}(i)$ attained from i. The maximum number of links $N_{\rm out}(N_{\rm out} - 1)$ normalizes the expression for $C(i)$. The object of interest is the mean $\langle C\rangle$. As usual, it is meaningful to compare the mean clustering coefficients of two distinct network reconnection modes. For the network with randomized $\overline{I}_{\rm out}$ links we obtained $\langle C_{\rm rand}\rangle \simeq 0.02$, while $\langle C\rangle \simeq 0.55$ ($\langle C\rangle/\langle C_{\rm rand}\rangle \simeq 27.5$) reached by $\hat{U}_{\rm Re}$ is much higher and thus empirically more relevant [43]. We have computed the average of the minimum path way for the partially random net $\langle l_{\rm min,rand}\rangle \simeq 2.9$ (with the circular subgraph untouched). The action of $\widehat{U}_{\rm Re}(i_{\rm r})$ provides $\langle l_{\rm min}\rangle \simeq 15.4$ however much smaller maximum $l_{\rm min,max} = 3$ of pdf($l_{\rm min}$). The combination of above topological attributes supports both small-world and scale-free concepts of network statistics. Fig.5 indicates that **RRW**(.,.,.) invokes generation of modular topology of several local densely connected leaders.

5. Conclusions

In this paper we reveal some interesting properties of the model of spin dynamics on a complex evolving network. We demonstrated that our model is relevant for the description of stock market statistics. The stationary regime is formed due to balance between information entropy inflow produced by the extremal dynamics

and stochastic outer sources compensated by the entropy outflow caused by the adaptive moves.

We are conscious that minority game term $-c_0 S^{(t)}(i) m^{(t)}$ of fitness only roughly describes the financial profits at the real stock markets. In further research we plan the comparison with \$-game model [44] that represents a step closer to stock market reality. Additional improvements of fitness could be done by considering the impact of regulatory and non regulatory halts and delays.

The most emergent aspects of agent systems near the criticality are the power-law distributions of the topological and spin characteristics that occcur as a consequence of self-organization processes. The necessary remark in this context is that exponents of such dependencies are non-universal, i.e. they can vary from one set of free parameters to another.

The adjustment of parameters can be formulated as multi-objective optimization computationally demanding task. Due to its comprehensive features, the problem is planned to be discussed in future works.

The site http://158.197.33.91/%7Ehorvath/selfstructured_net/ provides our simulation C++ code and supplementary materials.

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